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High-Speed RDC Data Averaging Through Dynamic Time Warping

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Abstract

High-speed diagnostics are essential for understanding the unsteady flow fields in rotating detonation combustors (RDCs). However, the experimental data from RDCs often exhibit significant stochasticity due to factors such as lap-to-lap detonation wave fluctuations, measurement uncertainties, and sensor-induced artifacts. Traditional phase-averaging techniques, like the arithmetic mean, can distort the true detonation wave structure by smoothing out key features due to temporal misalignment. This study investigates the application of soft-Dynamic Time Warping (soft-DTW) based averaging as a more accurate method for processing high-speed RDC data. Unlike conventional methods, soft-DTW is resilient to local time-axis distortions, allowing for better alignment and preservation of the intrinsic wave structure. The study evaluates the capability of soft-DTW to capture essential physical characteristics of rotating detonation waves using dynamic pressure and video data. Additionally, a sensitivity analysis assesses the method's effectiveness in accurately representing secondary features, such as reflected shock waves, highlighting its potential for more representative RDC data averaging.

Keyword: *Rotating Detonation Engine, Dynamic Time Warping, Pressure Gain Combustion*

1. Introduction

Rotating detonation combustors (RDCs) are a focus of pressure gain combustion (PGC) research, operating with continuously propagating detonation waves that increase stagnation pressure. These waves rotate within the combustion chamber at frequencies of several kHz. Experimental studies rely on high-frequency pressure data and diagnostics to estimate wave frequency and analyze detonation wave behavior [4, 3, 2, 8, 7]. Averaging time series data is crucial for understanding RDC wave dynamics but is complicated by high stochasticity in experimental data, due to factors like sensor noise, response time, and sampling rate. Traditional averaging methods, such as the Euclidean approach, often misrepresent the detonation wave structure by smoothing out sharp features and introducing alignment errors. For instance, Bohon et al. [4] used Euclidean phase averaging, which resulted in the loss of critical wave details due to lap-to-lap fluctuations. In contrast, the soft Dynamic Time Warping (soft-DTW) method offers a more accurate approach to averaging, preserving wave features. This paper outlines a detailed procedure for applying soft-DTW to RDC data, aiming to enhance its reliability and performance, especially compared to the commonly used Euclidean method. Further investigation is required to validate and optimize soft-DTW applied to RDC-type data, focusing on the key parameters that influence its performance.

2. Methods

2.1. Averaging of Time Series

The goal of time series averaging is to derive a representative time series, denoted as $\mathbf{x}$, that approximates a set of time series $\mathbf{Y} = \mathbf{y_1}, \mathbf{y_2}, \ldots, \mathbf{y_N}$ using a chosen cost-alignment metric. This resulting time series $\mathbf{x}$ is known as the barycenter. In this work each $\mathbf{y_j}$ in $\mathbf{Y}$ represents the pressure trace over a single detonation wave period for a detonation process characterized by the formation of a single wave. A common method for averaging is to minimize the Euclidean distance between $\mathbf{x}$ and the time series in $\mathbf{Y}$. However, the Euclidean distance is sensitive to temporal shifts, leading to poor alignment when time series have temporal fluctuations. To improve robustness, Dynamic Time Warping (DTW) is preferred, as it warps the time axis to align similar features, offering more accurate averaging [6]. Figure 1 highlights the difference between Euclidean and DTW distance in aligning two time series.

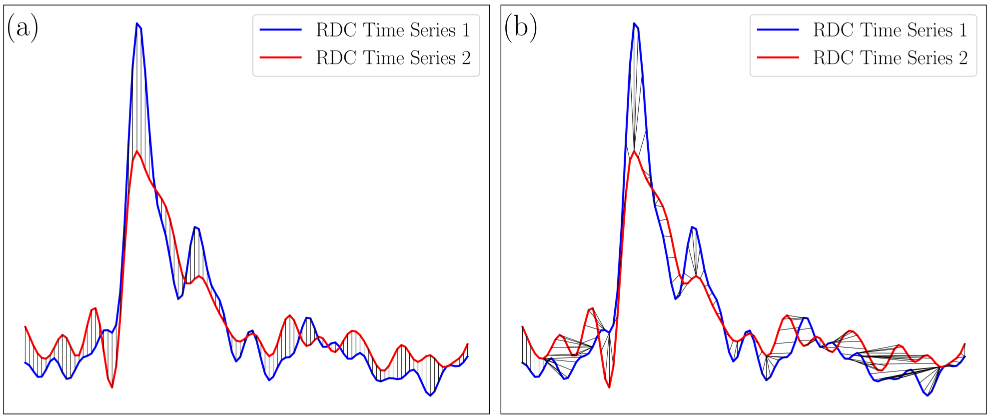

Fig. 1: Alignment of two example RDC time series data using (a) the Euclidean metric, and (b) the DTW metric.

The DTW time series alignment method selects the alignment with the lowest cost among all possible alignments [6]. In contrast, the soft-DTW method introduced by Cuturi et al. replaces the standard minimum operation with a "soft" minimum function [5]. Soft-DTW incorporates a smoothing parameter, γ, which accounts for the trade-off between smoothness and accuracy [**Blondel2021**]. Unlike the DTW method, soft-DTW evaluates all possible alignments of the time-series, assigning weights based on their probabilities according to the Gibbs distribution.

2.2. Experimental Setup

The data for this study were collected from a non-premixed rotating detonation combustor at TU Berlin, as described by Bach et al. [1]. The RDC has an annular chamber with a length of 110 mm, an outer diameter of 90 mm, and a gap width of 7.6 mm. Hydrogen is injected axially through 100 holes, while air is introduced radially inward, creating a jet-in-crossflow. Pressure measurements were acquired using PCB 112A05 piezoelectric transducers mounted along the combustor to capture dynamic pressure variations within the detonation zone and the oblique shock region. To protect the sensors, the runtime was limited to 300 ms, with data sampled at 500 kHz. The air mass flow rate was 500 g/s, and the global equivalence ratio was set to stoichiometric for optimal detonation.

2.3. Application of the soft-DTW Method on RDC Type Data

The primary objective of this study is to provide a detailed guideline for applying the soft-DTW method to average time series data from RDCs. The pressure signals obtained during experimental tests were processed and aligned using a DTW-based iterative averaging technique. Figure 3 presents the steps and key parameters required to capture the detonation wave dynamics. The flowchart differentiates between critical parameters explored in this study (highlighted in blue) and those established in the literature (highlighted in orange), which were not further examined.

The procedure consists of several key stages: post-processing of raw pressure data, configuring soft-DTW parameters, and performing barycenter calculations. Initially, pressure signals - either raw or filtered - are segmented into time series representing individual detonation wave cycles, determined by the average wave frequency (see Fig. 2). The time series are then aligned based on specific criteria, such as pressure slope alignment or peak alignment, which emphasizes the detonation wave's passage. Once aligned, the time series are subjected to time normalization according to the average wave period and pressure normalization between $[-1, 1]$. Key parameters governing the soft-DTW method include the number of time series, initialization, number of iterations, smoothing parameter, and weighting. The first two - time series selection and

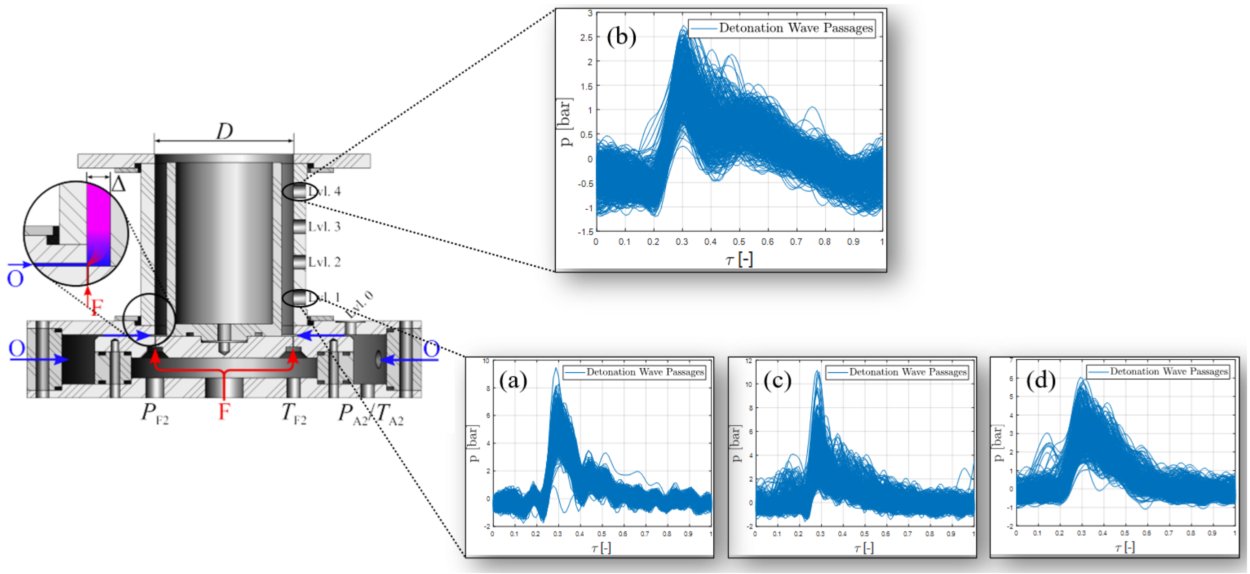

Fig. 2: RDC sensor locations and corresponding time series.

initialization - are critical as they influence the pressure peak's shape and timing in the calculated barycenter and are thoroughly analyzed in subsequent sections. The number of iterations primarily influences the convergence time of the calculation, with a relatively minor impact on the final barycenter compared to other key parameters. Cuturi et al. [5] recommend using a minimum of 50 iterations to ensure robust results. The smoothing parameter controls idiosyncrasies and noise present in the pressure time series; excessive smoothing may yields an underestimation of the wave peak, reducing accuracy. Weights were uniformly assigned, treating all time series equally in determining the mean wave structure. The process is divided into two stages: initialization and barycenter computation. Initializing the soft-DTW process with a precomputed barycenter mitigates the effects of time series stochasticity by starting the calculation with a barycenter that approximates the final shape. This approach enhances the accuracy of the final barycenter in representing the detonation wave dynamics and minimizes distortions introduced by anomalous trends in the data.

3. Results

3.1. Sensitivity to Soft-DTW Inputs

The soft-DTW method, as illustrated in Figure 3, is critically dependent on several key parameters that dictate the final outcome. The selection of these parameters is closely linked to the characteristics of the raw pressure time series obtained from the post-processing of experimental data. While variations in data quality may require adjustments to the setup, general procedures for optimal parameter selection can be established, enabling their application across various types of RDC datasets.

3.2. Alignment Criteria

Time series alignment is performed during the post-processing of pressure measurements. While various alignment criteria can be arbitrarily applied, in the context of analyzing the structure and dynamics of detonation waves using high-speed diagnostics, the focus typically lies on aligning wave passages with respect to characteristic features of the pressure peak. Two widely used alignment methods for RDC time series are based on the pressure peak and the slope of the pressure rise. In the first method, time series are aligned according to the maximum pressure value captured by the instrumentation, ensuring that the mean position of the barycentric peak is located at the same temporal point across all series. In contrast, the second method aligns wave periods based on the pressure gradient, which reflects the rate of pressure increase induced by the detonation. This approach overlaps the time series according to the time variation in pressure.

A comparison of these two alignment methods is presented in Figure 4. Both barycenters were computed using the procedure detailed in previous sections, considering the same time series and applying identical soft-DTW configurations. As shown, in both cases, the pressure peak is captured accurately, reflecting the expected sharp rise and subsequent expansion of the pressure. However, when aligning based on the peak value, a pronounced knee appears, unrelated to the detonation process, which corresponds to a strong positive

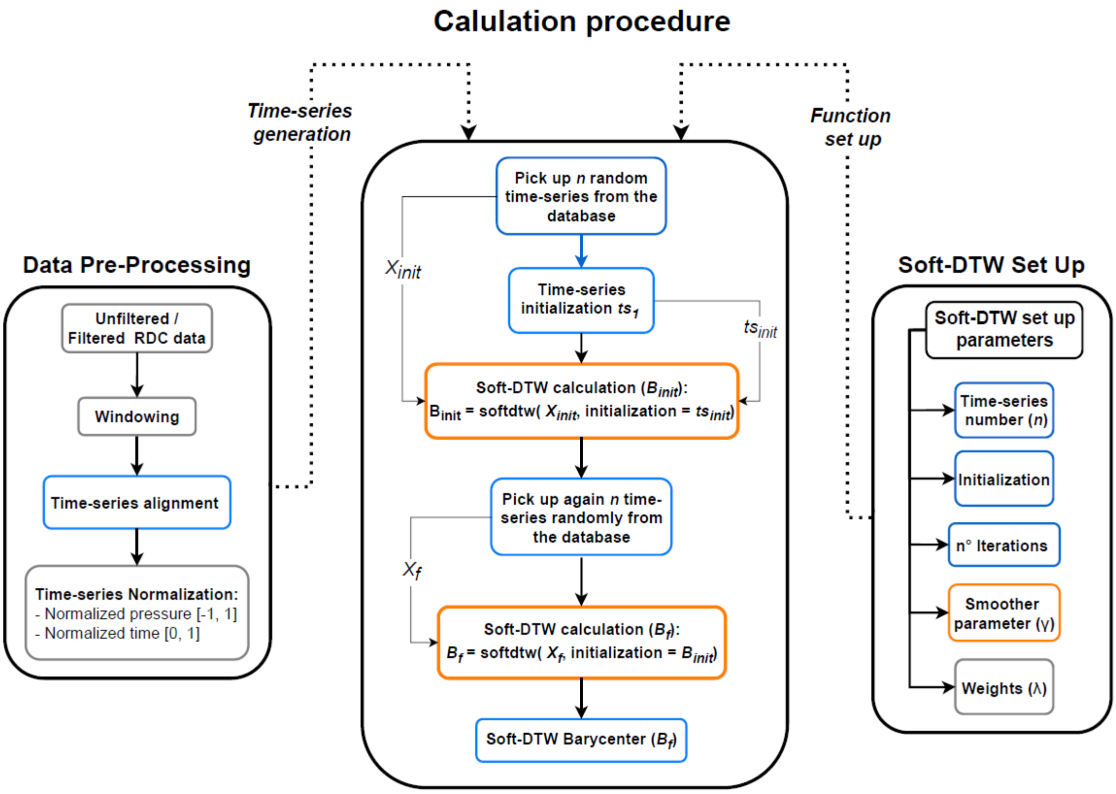

Fig. 3: Procedure for implementing the soft-DTW averaging method for RDC type time series.

pressure gradient. The rate at which the maximum pressure is reached, determined by the detonation reaction time, is a critical parameter for reconstructing wave dynamics. Therefore, aligning based on the pressure gradient of the rising edge provides a more balanced representation, capturing both the pressure peak and the underlying wave dynamics more effectively.

3.3. Initialization

The initialization used to calculate the barycenter via the soft-DTW method has been identified as a critical parameter with significant influence on the final barycenter's shape. Since the initializing time series or barycenter directly influences the resulting barycenter, careful selection of this parameter is important to increase the probability of converging toward an average that better reflects the structure of the time series under consideration.

Cuturi et al. [5] compared soft-DTW barycenters obtained using Euclidean initialization and a randomly selected time series from the analyzed dataset. Their results demonstrated that initializing with a random time series yields a barycenter that better preserves the structural integrity of the selected time series. However, due to the high degree of stochasticity in experimentally obtained RDC data, random selection of a time series for initialization is not feasible. A randomly chosen series may deviate significantly from the others in terms of peak shape, position, and intensity, resulting in a distorted barycenter that does not accurately reflect the average pressure values over the wave time period. To address this issue, a method to guide the selection of initialization has been proposed, minimizing the unpredictability associated with random selection while avoiding the limitations of Euclidean initialization. This approach ensures a more representative and robust barycenter that better conforms to the underlying characteristics of the time series data.

Given the critical importance of initialization shape in determining the final barycenter, the proposed procedure aims to identify an optimal time series for initialization. The primary objective is to find a time series that best approximates the entire dataset used for averaging.

Figure 5 illustrates the calculation sequence. The first step involves constructing the DTW Distance Matrix, which is an $n \times n$ symmetric matrix, where each element (i, j) represents the DTW distance between the i-th and j-th time series. The DTW distance quantifies the minimum alignment cost between two time series, considering point-wise differences between aligned values. Unlike Euclidean distance, DTW accounts for

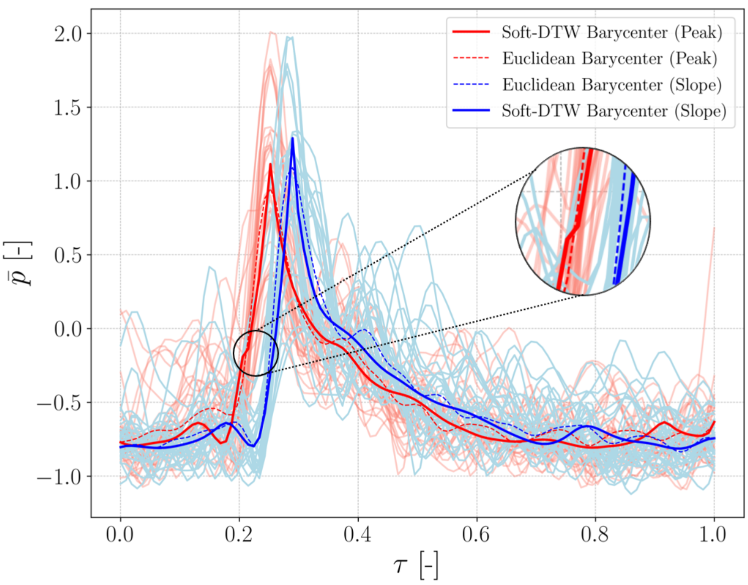

Fig. 4: Comparison of peak- and rising edge-aligned barycenters.

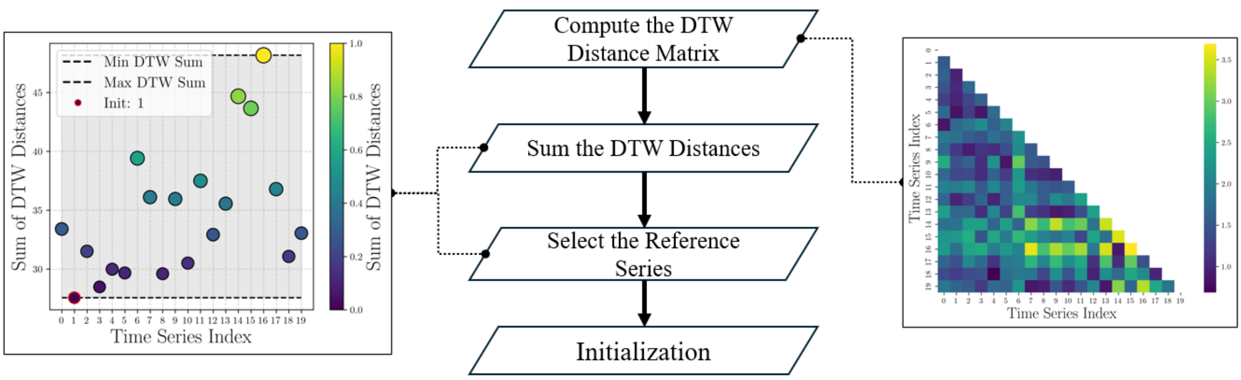

Fig. 5: Initialization procedure to identify the most representative individual time series.

nonlinear temporal shifts, making it particularly useful for comparing sequences that may be misaligned or vary in length. Once the DTW Distance Matrix is computed, the next step is to sum the distances between each time series and all others. The time series with the smallest cumulative distance relative to the rest of the dataset is considered the closest to the overall dataset in terms of DTW distance. This suggests that its shape and temporal alignment are already more representative of the barycenter, making it an ideal starting point for the soft-DTW barycenter calculation. To assess the robustness of the proposed method, the approach was applied to multiple RDC datasets, acquired from pressure sensors placed at distinct locations relative to the detonation zone and under varied operating conditions. This comprehensive evaluation across diverse experimental conditions ensures the method's applicability and reliability in capturing the underlying dynamics of detonation processes.

Figure 2 presents the wave periods for the various cases analyzed, post-processed using the procedure outlined in the flowchart. Specifically, three of the datasets were obtained from piezoelectric pressure sensors located directly in the detonation zone, where wave formation occurs, while the final dataset was derived from pressure measurements in the oblique shock region.

For each dataset, the method for determining initialization, as previously described, was applied. Figure 6 illustrates the results of the soft-DTW barycenter calculations using the proposed initialization procedure. For each dataset, the soft-DTW barycenters, Euclidean barycenter, and the time series selected for initialization (the 'output' of the initialization procedure) are shown. The overlay of the three barycenters with the time series used for calculation enables a direct comparison between the methods by highlighting both the dynamic and static characteristics of the detonation wave.

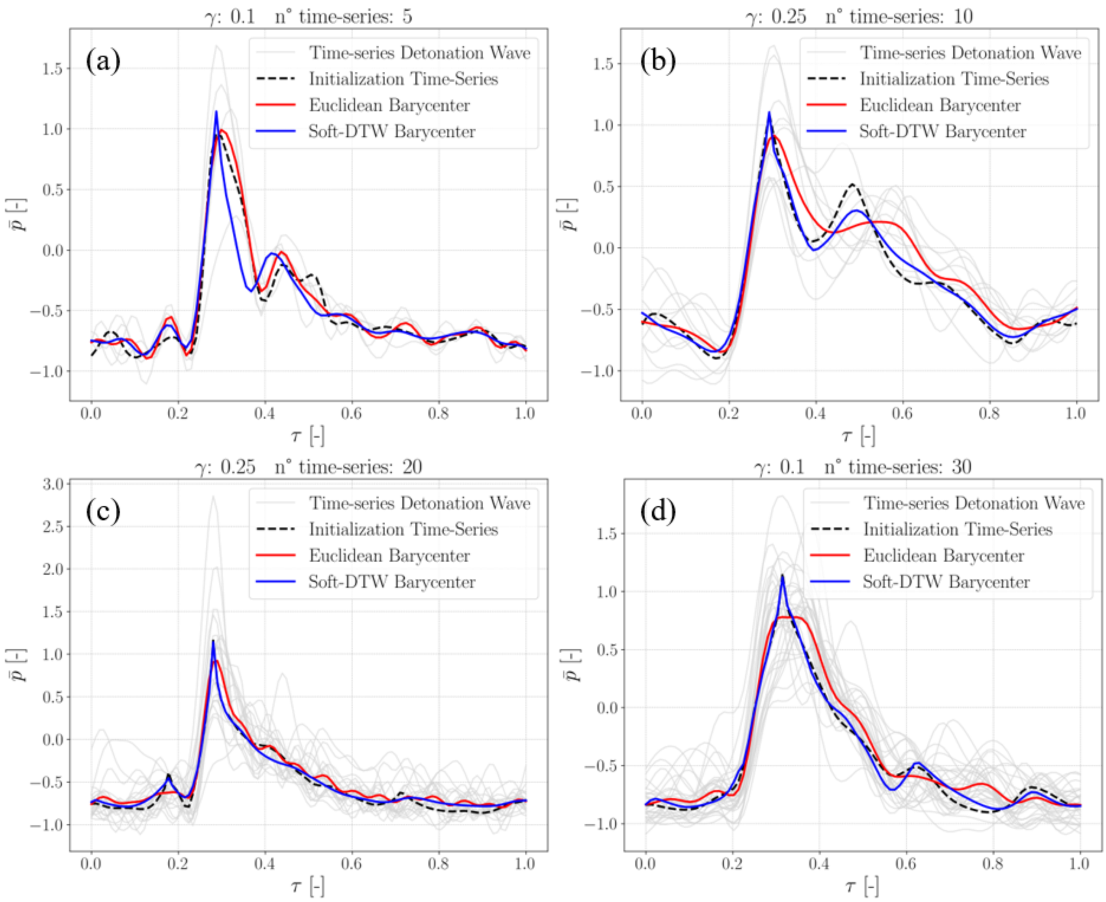

Fig. 6: Initialization procedure application on RDC data sets.

In all analyzed cases, the soft-DTW method demonstrates a superior ability to capture the trends in the raw time series, and the initialization aligns consistently with the final barycenter shape. This results in a more precise representation of the pressure peak, characterized by higher sharpness, and of the time pressure rise, identifiable by the positive slope of the peak. The soft-DTW method maintains consistency across all wave transitions, accurately reflecting the experimental measurements. This confirms both the feasibility and robustness of the computational sequence employed for determining the initialization.

The primary shortcoming of the Euclidean barycenter is clearly visible in Figures 6(b) and (d). In cases where the experimental data exhibit greater dispersion, the Euclidean barycenter excessively smooths the pressure peaks, producing a non-physical temporal pressure trend. In Figure 6(b), the Euclidean barycenter fails to capture the distinctive shape of the time series, particularly when a double peak occurs over a short temporal interval, further highlighting its limitations in resolving complex wave dynamics.

3.4. Dependence on the Number of Time Series

The number of time series considered is closely tied to the quality of the data measured by the instrument. Significant variability among time series can lead to underestimation of both the pressure peak and the rate at which it is reached. However, the soft-DTW method mitigates these challenges, effectively handling the stochasticity present in the pressure signals.

Figure 7 compares barycenters generated by the two methods for two distinct cases. In case 7(a), most time series exhibit pressure peaks within a narrow temporal range, whereas in case 7(b), the peak positions are more variable and dispersed. The Figure 7 highlights the advantages of the soft-DTW method, showing that the number of time series considered has minimal impact on the resulting barycenter, as evidenced by the nearly identical overlapping barycenters. In case 7(a), the Euclidean barycenter also exhibits minimal dependence to variation in the number of time series; however, this does not hold true for case 7(b). The broader temporal distribution of peak pressure in case 7(b) causes the Euclidean method to excessively smooth the signal, significantly underestimating the peak. Nevertheless, increasing the number of time series in the Euclidean method reduces this underestimation and enhances peak sharpness, bringing it closer to the true time series dynamics.

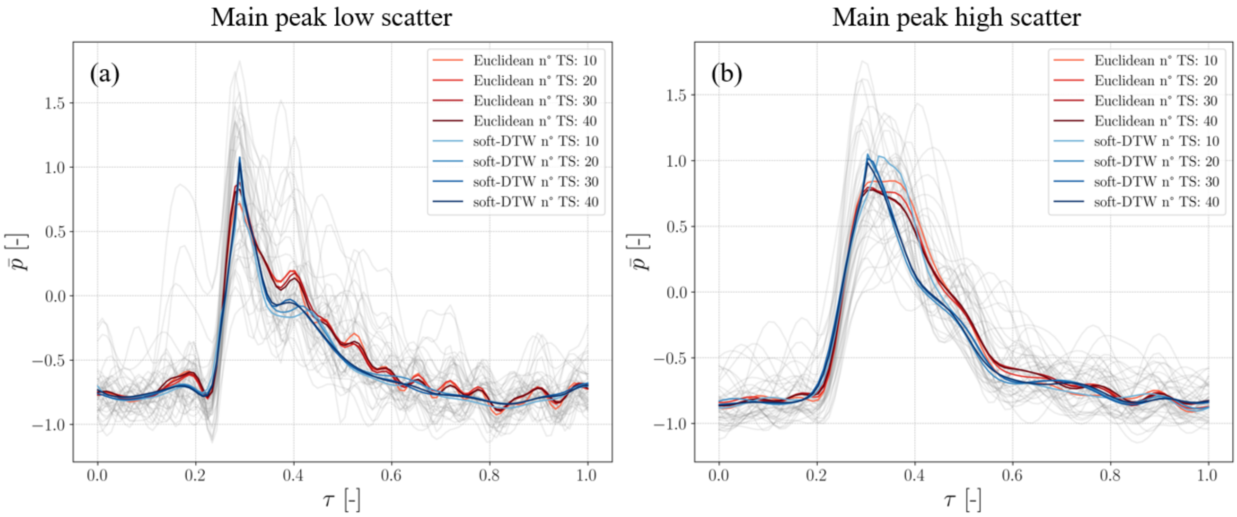

Fig. 7: Comparison of soft-DTW and Euclidean methods across different numbers of time series.

4. Conclusion

This paper presents a procedure for averaging RDC type pressure time series using the soft-DTW method. From defining the time series to configuring the key parameters of the soft-DTW calculation, the procedure establishes a clear framework for obtaining reliable results from RDC type data. The analysis demonstrates that the soft-DTW method is more effective than the Euclidean approach in constructing the average circumferential wave structure. Specifically, it allows for consistent extraction of the wave peak and its dynamics, even in the presence of considerable fluctuations in wave velocity between cycles. Furthermore, the importance of proper initialization is highlighted, as it facilitates the generation of a barycenter that closely aligns with the underlying wave dynamics, ultimately improving the accuracy and robustness of the analysis.

Acknowledgments

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