Vorticity and Enstrophy transport in rectangular supersonic jets

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Abstract: Vorticity dynamics is an important area in fluid dynamics due to the associated scales of vortices from largest to smallest in energy cascade process. Enstrophy is another measure of vorticity which is computed as mean square fluctuating vorticity and is linked to the dissipation of turbulence kinetic energy. Both quantities play a crucial role in compressible turbulence where flow discontinuities such as shock waves are present as they may contribute to the vorticity generation. High-fidelity Large Eddy Simulations (LES) are conducted to assess vorticity and enstrophy transport in supersonic rectangular jets at over- and under-expanded conditions. The underlying terms in vorticity transport equation are analyzed for their contributions in vorticity budgets. It is shown that vortex stretching is the dominant term in vorticity budget across the shear layer whereas baroclinic effects are pronounced across a Mach wave in the core of the jet due to the misaligned gradients of density- and pressure.

I. INTRODUCTION

Turbulence in compressible flow is an important area of research due to its wide applicability in aerospace science such as in supersonic jets, combustion processes and shock boundary layer interactions. It feeds on energy transfer from mean flow to turbulent fluctuations and it dissipates at the smallest scales. During this process, fluid particles experience stresses induced by momentum fluxes also known as Reynolds stresses. Vorticity transport is an important area in fluid dynamics since it is linked to the process of energy cascade where large scale eddies transfer their energy to the smaller scales and then dissipation occurs at smallest scales. This process can be better analyzed by directly assessing the vorticity transport equation as vortices exist over a range of scales and the transport equation gives access to all underlying mechanisms. Vorticity transport is directly affected by fundamental flow regime – whether the flow is subsonic, or supersonic and/or incompressible or compressible, as additional terms may arise depending on flow regime. For example, baroclinic term arises in compressible form of vorticity equation while it is absent in its incompressible form as the density is constant.

Over the last few years, rectangular jets have gained attention in the aerospace field. Jet flows are vortical in nature as they are characterized by shear layers and mixing region where different terms in vorticity transport equation are pronounced. Supersonic jets are also characterized by shock and expansion waves where the vorticity transport is affected by the dilatation of velocity vector field as well as due to the gradients of density- and pressure. In present work, the vorticity transport in rectangular supersonic jets is examined to address the key mechanisms for the generation of vorticity. Since current work deals with supersonic flow, compressible form of vorticity equation is used, as explained below. Vorticity transport equation can be derived by taking the curl of Navier-Stokes momentum equation and is shown below (1). Terms on LHS represent temporal and terms on RHS represent advection, dilatation, vortex stretching, baroclinic torque and dissipation.

$$\frac{\partial \omega}{\partial t} = -\underbrace{\mathrm{u}.(\nabla\omega)}_{advection} - \underbrace{\omega(\nabla.\mathrm{u})}_{dilatation} + \underbrace{(\omega.\nabla)\mathrm{u}}_{vortex\ stretching} - \underbrace{\frac{1}{\rho^2}\nabla p \times \nabla\rho}_{baroclinic\ torque} + E_\omega \tag{1}$$

Enstrophy is another measure of vorticity and is given as $\Omega = \omega_i\omega_i$. It is a scalar quantity. Below equation is the enstrophy transport equation in for compressible flow. Enstrophy transport equation can be derived by taking inner product of vorticity with vorticity transport equation. Terms on RHS in equation (2) are vortex stretching, viscous dissipation and baroclinic torque.

$$\frac{\partial Q}{\partial t} = -\underbrace{\nabla.(uQ)}_{advection} - \underbrace{Q(\nabla.u)}_{dilatation} + \underbrace{\omega.S.\omega}_{vortex\ stretching} - \underbrace{\frac{1}{\rho^2}[\omega.(\nabla p \times \nabla\rho)]}_{baroclinic\ torque} + E_Q \tag{2}$$

Several previous studies have assessed vorticity transport in multiple flow regimes. Some notable work by Kida and Orszag [1] sheds light on vorticity and enstrophy transport in supersonic mixing layers. They showed that vortex stretch is the dominant term followed by baroclinic torque which is pronounced across a shock wave. Cavo et al [2] showed enstrophy budgets in incompressible round jets using experimental testing. Their analysis showed that vortex stretching is approximately balanced by dissipation which was obtained as a balance of other terms. Apart from that, few studies [3,4] have assessed vorticity transport during combustion in supersonic cavity and during combustion process in scram jets. Peterson [4] noted that baroclinic torque was amplified by combustion while dilatation and vortex stretching are suppressed by combustion in supersonic cavity flame holder. Enstrophy has also been assessed in incompressible flow regime in a turbulent channel flow using DNS as described in [5]. Sinha [6] studied enstrophy amplification in highspeed flows while Miura [7] numerically investigated enstrophy generation in shock-dominated flows and noted that enstrophy is generated in the form of vortex tubes behind curved shocks. Cheprasov [8] evaluated the vorticity transport mechanisms in round supersonic jet using LES and noted that baroclinic effects were relatively small in the vicinity of the jet. These studies were either based on experimental testing (as in [2]) or based on numerical simulations employing DNS (as in [1,5,7]) or hybrid RANS-LES, i.e., IDDES (as in [3]) or based on LES (as in [8]). To authors' best knowledge, there have been no studies that characterized various terms in vorticity and enstrophy transport in supersonic rectangular jets using LES. Overall, there has been limited open-source literature that assessed vorticity and enstrophy transport simultaneously in high-speed jets and this paper attempts to bridge the gap.

II. METHODOLOGY

All simulations are performed using commercial solver Simcenter Star-CCM+ version 15.04.008-R8 [9] and use WALE subgrid scale model suggested by Nicoud and Ducros [10]. The rectangular nozzle geometry is shown in Figure 1. The LES uses 73 million grid cells with appropriate refinement zones in the nozzle internal region, jet nearfield and plume region. Note that one of the challenges in quantifying the budget terms is ensuring convergence of budget statistics. This typically requires the simulations to be run over several flow-through-times which increases the computational time and cost.

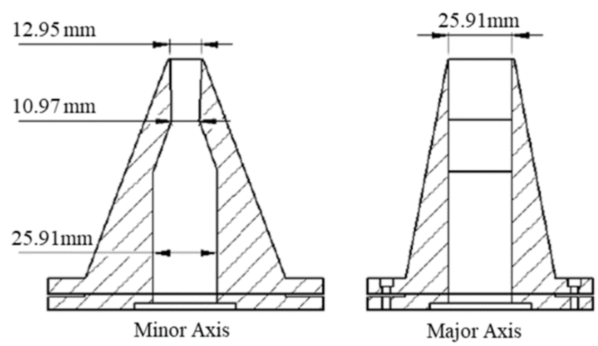


Figure 1. Rectangular nozzle geometry.

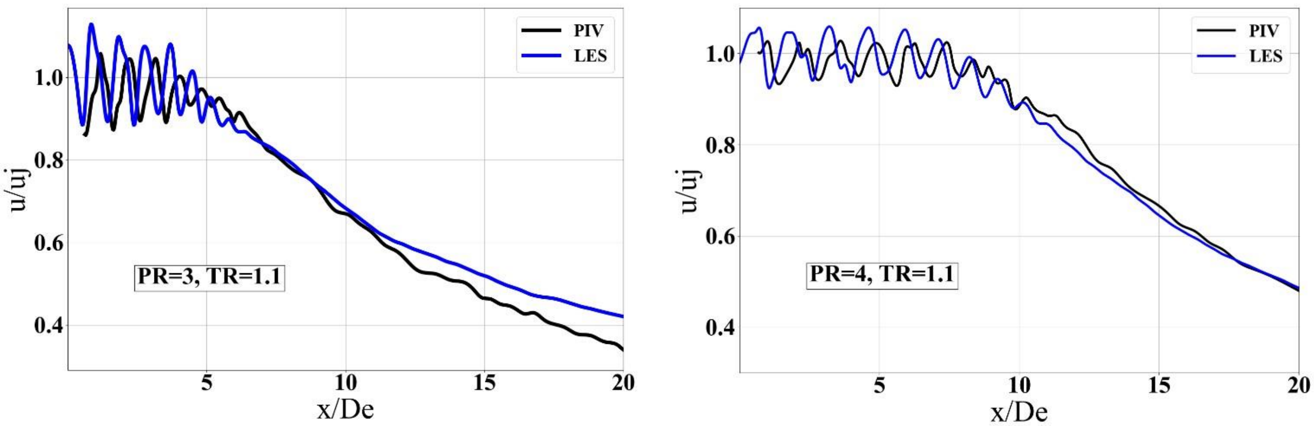
A. Boundary conditions

TABLE 1. Operating conditions for LES cases.

Flow condition	*Pressure ratio*	*Temperature ratio*	*Jet-exit Mach number*	*Jet-exit velocity*
Over-expanded	3	1.1	1.35	422
Under-expanded	4	1.1	1.55	465

B. Validation

Figure 2 shows the validation plots for both flow conditions. An important detail here is the location and amplitude of shock cells in experimental data and numerical results. Although LES matches closely, the location and amplitude of shock cells are not exactly coincident with experimental data. Several factors influence this behavior, resulting from both experimental techniques as well as numerical techniques. While PIV is a non-intrusive technique, it suffers from various uncertainties resulting from equipment such as the spatial resolution, processing algorithm, and particle lag.

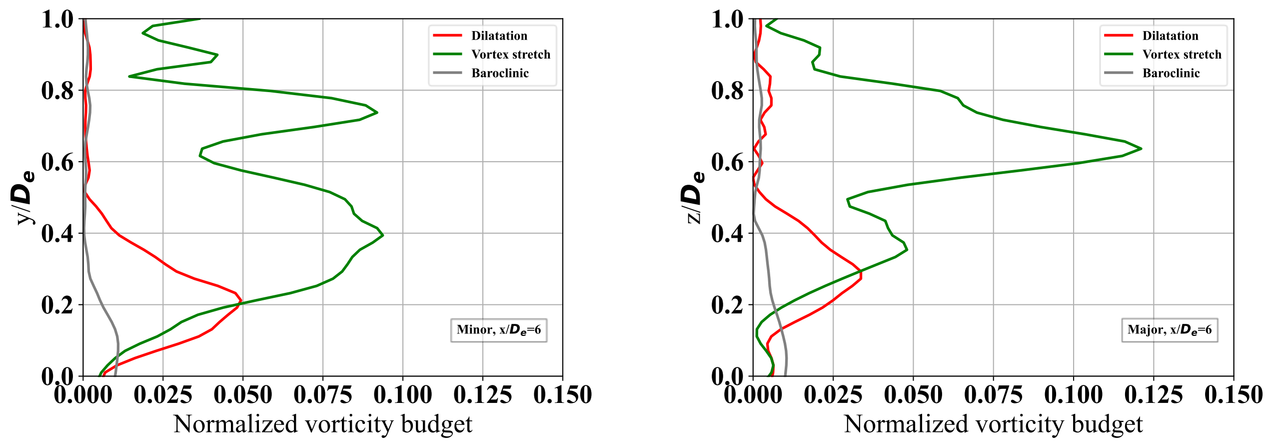


(a) Over-expanded case (b) Under-expanded case

Figure 2. Jet centerline velocity compared against PIV.

III. RESULTS

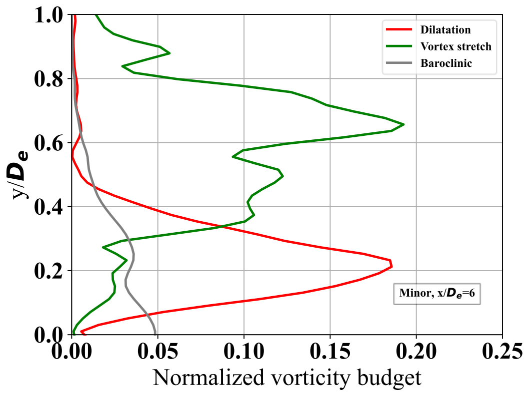
This section analyzes the results of vorticity budgets for both flow conditions on minor and major axis planes. As seen from Figure 3, vortex stretching is the dominant term, followed by dilatation term. Divergence of velocity vector contributes to the overall magnitude of dilatation term and in turn generates vorticity. Baroclinic term arises due to the misaligned gradients of density and pressure and since Figure 3 shows the budgets across the jet shear layer, baroclinic term is non-zero towards the jet centerline. This is because the jet core experiences shock- and expansion waves where the gradients of density and pressure jump steeply and contribute to vorticity.



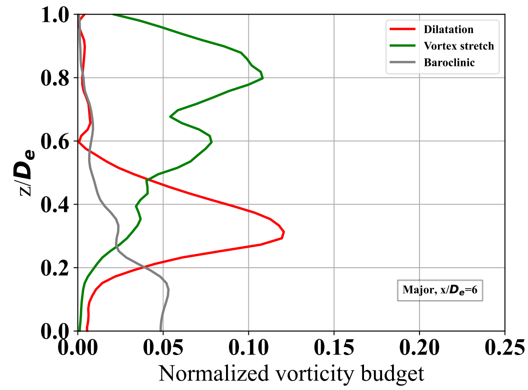
(a) Minor axis (b) Major axis

Figure 3. Normalized vorticity budget magnitudes for over-expanded case at x/De = 6 – dilatation, vortex stretching and baroclinic terms.

Figure 4 shows the normalized vorticity budget magnitudes for the under-expanded case across the jet shear layer at x/De = 6. As seen, vortex stretching is dominant towards the edge of the jet while dilatation and baroclinic terms are dominant towards the core of the jet. The magnitudes of these terms are higher than that of the over-expanded case.



(a) Minor axis

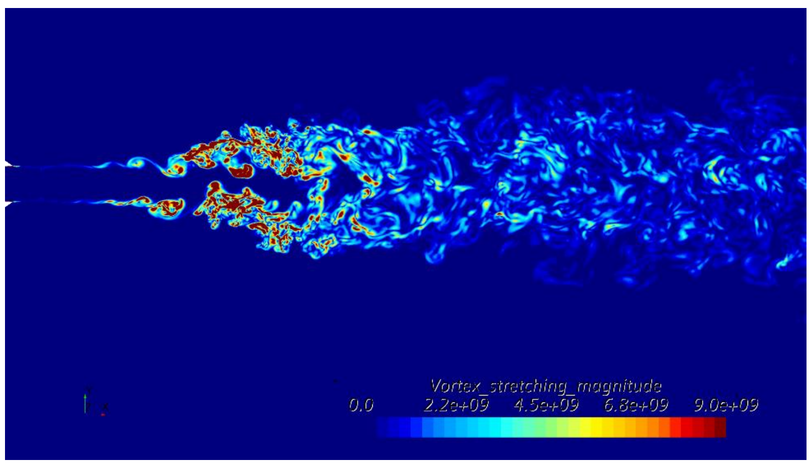


(b) Major axis

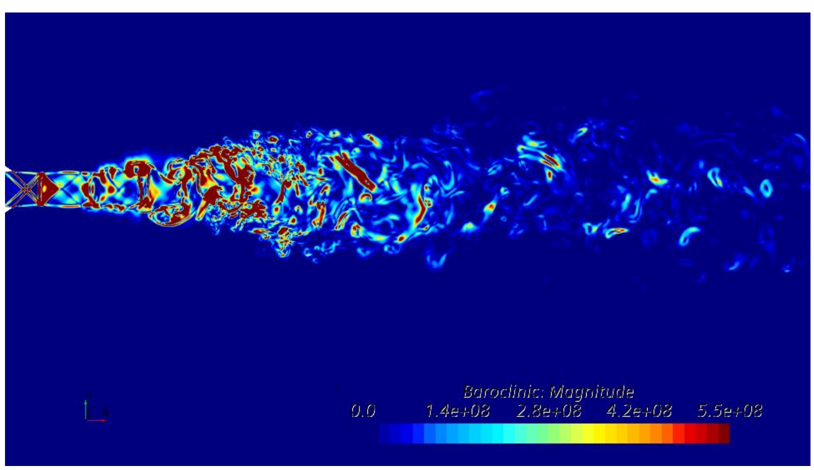
Figure 4. Normalized vorticity budget magnitudes for under-expanded case at x/De = 6 – dilatation, vortex stretching and baroclinic terms.

IV. FLOW VISUALIZATION

This section shows the contour plots of various vorticity budget terms. Vortex stretching is dominant in the shear region while baroclinic term is predominantly active in the jet core.



(a) Vortex stretching



(b) Baroclinic term

Figure 5. Instantaneous vorticity budget magnitudes for over-expanded case.

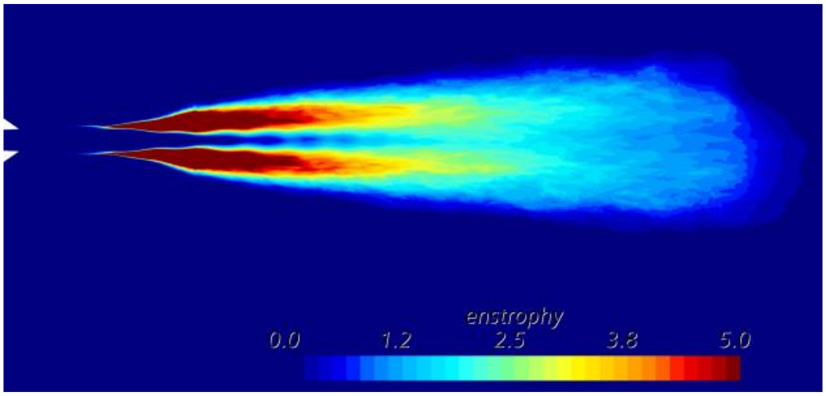


Figure 6. Normalized enstrophy contour for under-expanded case.

IV. CONCLUSIONS AND FUTURE WORK

This paper addresses vorticity and enstrophy transport in rectangular supersonic jets at over- and under-expanded flow conditions using LES. The LES database is first validated with experimental measurements conducted using PIV. They are found to be in good agreement with PIV. Then, the vorticity budget terms are examined for their contributions in various regions of the jet nearfield. It is shown that vortex stretching term is the dominant term in the shear region while dilatation and baroclinic terms are active predominantly towards the jet core. Enstrophy is another measure of vorticity and can be though of as rotational turbulent kinetic energy. Enstrophy can be linked to TKE dissipation and is active in the jet shear layer. Finally, this paper presented the ongoing work on vorticity and enstrophy transport and as part of continued efforts, more details are being analyzed.

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