Quantitative Flow Visualization of a Square Underexpanded Microjet by RST

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Abstract

Underexpanded microjet issuing from a square Laval nozzle of a design Mach number of 1.5 is quantitatively visualized by rainbow schlieren tomography. The test nozzle has a channel area of $1 \text{ mm} \times 1 \text{ mm}$ at the exit, and an equivalent diameter of 1.13 mm. Flow visualization is performed at a nozzle pressure ratio of 7.0 to produce strong shocks in the jet plume. The Reynolds number based upon the equivalent diameter and flow properties at the nozzle exit is 9.37×10^5 . Multi-view rainbow schlieren pictures with the horizontal filter setting are taken by rotating the nozzle around its central axis at an equal interval of 5° ranging from 0 to 180°. The jet density field is reconstructed by the convolution back projection method. The flow features of shock-containing square jets are demonstrated.

Keywords: Square Laval nozzle, Rainbow schlieren tomography, Shock wave, Supersonic jet.

1. Introduction

In recent years, there has been considerable research on the subject of the dynamics of a supersonic microjet for the application of microscale devices, including a critical nozzle for obtaining mass-flow rate at a low Reynolds number, a micropump based on the principle of macroscopic vapor-jet and diffusion-pump, a small satellite thruster in space engineering, gas wiping nozzles for continuous galvanizing line, etc. A deep, detailed comprehension of the flow characteristics through such devices requires information regarding the quantitative velocity, density, and temperature measurements in the flow field. However, even for supersonic jets from macroscale round nozzles, the jet characteristics are almost only qualitatively known [1].

Although the schlieren technique is well known as a typical visualization method for compressible flow, it is generally capable of examining the flow field only qualitatively. This is because it captures the density gradient in the direction parallel or perpendicular to the flow as two-dimensional information averaged along the line-of-sight direction. Therefore, complex integral calculations with special algorithms are required to obtain the three-dimensional density of the flow field using the schlieren method. In recent years, however, with the advent of computers with greatly improved performance and storage capacity, quantitative visualization methods that were previously difficult to obtain have been proposed.

Nazari et al. [2] experimentally investigated instantaneous three-dimensional structures of microjets issuing from round and square Laval nozzles operating under design and off-design conditions. They quantitatively measured the three-dimensional density fields of the microjets using 20 schlieren optical systems placed at equal intervals around the nozzle. Tashiro et al. [3] systematically investigated the shock structure and topology of underexpanded microjets from a round convergent nozzle using rainbow deflection deflectometry. To the best of the authors' knowledge, there are no studies on the three-dimensional structure of microjets emerging from square nozzles, except for the work of Nazari et al. [2]. In the present study, the three-dimensional structure of an underexpanded square microjet is experimentally investigated using the rainbow schlieren tomography (RST) [4].



Fig. 1: Schematic diagram of the experimental apparatus with rainbow schlieren optical system



Fig. 2: Schematic diagram of a test nozzle with a coordinate system.

2. Experimental Methods

Experiments are conducted in an intermittent blowdown compressed-air facility. Ambient air is pressurized by a compressor up to 1 MPa and stored in a high-pressure reservoir comprising two storage tanks with a total capacity of 2 m³ after filtering and drying. The high-pressure dry air enters a plenum chamber through the coupling shown in Fig. 1, reaches a stagnation state. It is then discharged into the quiescent air after accelerated to supersonic speed through a test nozzle that can be rotated about its central axis [4].

As shown in Fig. 2, a Laval nozzle with a square cross-section of 1 mm height by 1 mm width at the exit and a design Mach number of 1.5 is used as a test nozzle. The upper and lower wall contours from the nozzle inlet to the throat are designed by a sinusoidal curve to provide uniform flows at the nozzle inlet and throat. In addition, the upper and lower wall contours from the throat to exit are designed based on the method of characteristics to provide smooth uniform flows at the exit. The spanwise width of the nozzle is held constant



Fig. 3: Calibration curve of a rainbow filter

at 1 mm over a full length of 8.32 mm in the flow direction. The equivalent diameter at the nozzle exit, which is given by $D_{eq} = (4/\pi)^{1/2}$ for the present nozzle, is approximately 1.1 mm where the D_{eq} is defined as the diameter of a virtual circular nozzle with a cross-sectional area equal to that of the actual nozzle [5]. As shown in Fig. 2, a Cartesian coordinate system (x, y, z) is used, with the origin 'O' at the center in the cross-section of the nozzle exit, the 'x' axis in the horizontal direction, the 'y' axis in the vertical direction (perpendicular to the x axis), and the 'z' axis in the flow direction. In addition, we refer to the yz-plane including the z-axis as the symmetric plane and the plane inclined 45 degrees from the symmetric plane as the diagonal plane. As shown in Fig. 2, the t-axis is taken perpendicular to the z-axis along the diagonal plane. Furthermore, we refer to the observation of a flow field perpendicular to the symmetry plane as the symmetry plane view and that perpendicular to the diagonal plane as the diagonal plane view.

The nozzle pressure ratio (NPR), which is defined as the ratio of the stagnation pressure (p_{os}) upstream of the nozzle to the back pressure (p_b) is achieved by changing the p_{os} , with the p_b held constant at NPR = 7.0 to produce an underexpanded jet with a fully expanded Mach number of $M_j = 1.93$. For the present nozzle, the theoretical nozzle pressure ratio to realize the shock free jet or the design nozzle pressure ratio is approximately 3.67 for $\gamma = 1.4$. In the present experiments, p_b is 102 kPa and the ambient temperature T_b is 297 K. The Reynolds number, based upon the equivalent diameter and flow properties at the nozzle exit, is 9.37×10^5 .

Quantitative flow visualization is performed by rainbow schlieren tomography (RST) [4]. A rainbow filter is installed parallel to the flow in the cutoff plane of the schlieren. The filter is calibrated immediately before the experiment. The calibration curve of the rainbow filter used in the experiment is shown in Fig. 3. The calibration curve is a fifth order least-squares approximation. The reconstruction of the jet density field is carried out by the convolution back projection method. The rainbow schlieren pictures of the microjets are taken using a digital camera (Nikon D7100, 6000×4000 square pixel resolution with a 14-bit pixel depth) with an exposure time of 1/400 s and ISO of 1000 under a continuous schlieren light source (SIGMAKOKI, IMH-250).

3. Results and Discussion

3.1. Flow visualization

The rainbow schlieren pictures of a square underexpanded microjet for the symmetric and diagonal plane views are shown in Figs. 4(a) and 4(b), respectively. The schlieren picture are taken with the horizontal filter setting in the cut-off plane of the schlieren optical system. The jet boundary, expansion waves from the nozzle exit lip, and a Mach stem with reflected shocks can be seen in Fig. 4(a). Some shocks can also be recognized downstream of the Mach stem. The schlieren image of the diagonal plane view in Fig. 4(b) clearly shows the incident shock generated by the nozzle exit lip, the Mach stem with reflected shocks. Shocks intersecting

just after the Mach stem and what appear to be slip lines can also be faintly observed. Comparing the jet boundaries of the symmetric and diagonal plane views, it is clear that the former has a sharp radial spread immediately after the nozzle exit and then extends gradually in the flow direction, while the latter is almost constant from the nozzle exit to the downstream direction. Note that the schlieren pictures indicate the density gradients averaged in the line-of-sight direction, making it difficult to obtain details of the three-dimensional structure of the jet.



Fig. 4: Rainbow schlieren pictures of the square underexpanded jet for (a) symmetric plane view and (b) diagonal plane view where NPR = 7.0 and $M_j = 1.93$.



Fig. 5: Density contour plots of the square underexpanded microjet in the (a) symmetric plane and (b) diagonal plane.

3.2. Density contour plots

The density contour plots of the square underexpanded microjet in the symmetric and diagonal planes are represented in Figs. 5(a) and 5(b) with the flow from left to right. The density fields are normalized by the ambient density ρ_b . The contour level is illustrated as a color bar at the top of Fig. 5 over the range ρ/ρ_b 0.7 to 2.3 at intervals of 0.1. The red dashed line parallel to the abscissa indicates the lipline (y = 0.5 mm) in the symmetry plane and that (t = 0.73 mm) in the diagonal plane. The shock structure and topology inside the first shock-cell can be observed in detail. The jet boundary extends gradually from the nozzle exit to z = approximately 2 mm and then has a complex wavy distribution in the downstream direction. There is a trapezoidal region of low density just before the Mach stem. Immediately after the reflected shock, there are high density regions with a peculiar shape on both sides with respect to the central axis of the jet. The region surrounded by this unique shape has a sharp increase in density, and we refer to this boundary as the encircle shock in this paper. There is no strong shock downstream of the Mach stem, but instead a weak X-shaped 2nd shock appears. Compared to the symmetry plane, the jet boundary in the diagonal plane changes little in the flow direction. However, immediately after the reflected shock, a vertical shock can be observed that straddles the compression regions that exist on both sides of the jet's central axis. We call this vertical shock a straddle shock.

3.3. Streamwise density profiles

The streamwise density profiles along the jet centerline, lipline in the symmetric plane, and lipline in the diagonal plane are presented in Fig. 6. The dashed line parallel to the abscissa denotes the normalized fully expanded jet density ($\rho_j/\rho_b = 1.74$). Note that the fully expanded jet pressure (p_j) is equal to the back pressure (p_b) [6]. The left pointing arrow indicates the theoretical density ratio that can be derived based upon the assumption of the isentropic flow from the plenum chamber to nozzle exit. The density value



Fig. 6: Streamwise density profiles along the (a) jet centerline (x = y = 0), (b) lipline in the symmetric plane (x = 0 and y = 0.5 mm), and (c) lipline in the diagonal plane (x = 0.5 mm, y = 0.5 mm, t = 0.71 mm).

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immediately after the nozzle exit in Fig. 6(a) is considerably lower than the theoretical value due to the effect of light reflection at the cross-section of the nozzle exit. Since the NPR is 7 in the present experiment, the density along the central axis of the jet drops off abruptly up to z = about 1.6 mm from the nozzle exit. The streamwise location of the density minimum coincides with that of the Mach Stem. The flow Mach number just upstream of the Mach stem can be estimated as $M_1 = 4.0$ from the minimum density value because the flow is isentropic from upstream of the nozzle to just before the Mach stem [3]. For reference, Tashiro et al. [3] proposed an experimental relation

$$M = 1 + 0.72 \,(\text{NPR} - 1.89)^{0.76} \tag{1}$$

for the flow Mach number just before a Mach disk in an underexpanded microjet from a round convergent nozzle. Substituting the present experimental condition NPR = 7.0 into Eq. (1), we obtain M = 3.5.

The density along the central axis of the jet rises sharply due to the Mach stem, then decreases slightly, followed by a large rise and fall due to the straddle shock and post-shock expansion. The density variation across the second shock is almost the same as that across the first shock.

The density in the flow direction along the lipline in the symmetry plane expands rapidly from the nozzle exit, followed by a three-step rapid density increase due to an incident shock, reflected shock, and encircle shock, with the peak value considerably higher than the density increase due to the Mach stem. On the other hand, the density along the lipline in the diagonal plane decreases slowly in the flow direction since there are no shocks in the lipline.

4. Conclusions

An underexpanded microjet issuing from a square Laval nozzle with a design Mach number of 1.5 with an equivalent diameter of 1.13 mm at the exit was experimentally investigated by rainbow schlieren tomography for NPR = 7.0. The conclusions obtained are as follows.

(1) Straight incident and reflected shocks are formed in the first shock-cell, which shares its triple point with a downstream curved Mach stem. Immediately after the reflected shock, an encircle shock and a straddle shock are formed, causing the density to be higher than that immediately after the Mach stem.

(2) In the present experiment, the freestream Mach number $M_1 = 4.0$ just before the Mach stem becomes much higher than the fully expanded jet Mach number $M_j = 1.93$.

(3) The streamwise density along the lipline in the symmetry plane increases in three stages due to the incident shock, reflected shock, and encircle shock. On the other hand, the streamwise density along the lipline in the diagonal plane decreases gradually in the flow direction.

Acknowledgments

This work was supported in part by Institute of Environmental Science and Technology, The University of Kitakyushu.

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